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Tables for the Binary Sextic.

The Leading Coefficients of the First 18 *of the* 26 *Covariants.*

By Professor Cayley.

Including the sextic itself, the number of covariants of the binary sextic is $= 26$, as shown in the table p. 296 of Clebsch's "Theorie der binären algebraischen Formen," Leipzig 1872; viz. this is

	Order.						
Deg.	0	2	4	6	8	10	12
1				f			
2	A		i		H		
3		l		p	(f, i)		T
4	B		$(f, l)_2$	(f, l)		(H, i)	
5		$(i, l)_2$	(i, l)		(H, l)		
6	A_{ll}			(p, l) $((f, i), l)_2$			
7		$(f, l^2)_4$	$(f, l^2)_3$				
8		$(i, l^2)_3$					
9			$((f, i), l^2)_4$				
10	$(f, l^3)_6$	$(f, l^3)_5$					
12		$((f, i), l^3)_6$					
15	$((f, i), l^4)_8$						

Or, using the capital letters A, B, . . . Z to denote the 26 covariants in the same order, the table is

	0	2	4	6	8	10	12
1				A			
2	B		C		D		
3		E		F	$G, =(A, C)^1$		H
4	I		$J, =(A, E)^2$	$K, =(A, E)^1$		$L, =(D, C)^1$	
5		$M, =(C, E)^2$	$N, =(C, E)^1$		$O, =(D, E)^1$		
6	P			$Q, =(F, E)^1$ $R, =(G, E)^2$			
7		$S, =(A, E^2)^4$	$T, =(A, E^2)^3$				
8		$U, =(C, E^2)^3$					
9			$V, =(G, E^2)^4$				
10	$W, =(A, E^3)^6$	$X, =(A, E^3)^5$					
12		$Y, =(G, E^3)^6$					
15	$Z, =(G, E^4)^8$						

A is the sextic.
B " Salmon's A, p. 202.
I " " B, p. 203.
P is Salmon's C, p. 204.
W " " D, p. 207.
Z " " E, p. 253.
The references are to Salmon's Higher Algebra, 2d Ed., 1866.

In the present short paper I give the leading coefficients of the first 18 covariants, A to R (some of these are of course known values, but it is convenient to include them): for the next four covariants S, T, U, V, the leading coefficients depend upon the coefficients of A, C, G and E^2, viz. writing

$$\begin{aligned} A &= (a,\ b,\ c,\ d,\ e,\ f,\ g \between x,\ y)^6 \\ E^2 &= (\alpha,\ \tfrac{1}{4}\beta,\ \tfrac{1}{6}\gamma,\ \tfrac{1}{4}\delta,\ \varepsilon \between x,\ y)^4 \\ C &= (\alpha',\ \tfrac{1}{4}\beta',\ \tfrac{1}{6}\gamma',\ \tfrac{1}{4}\delta',\ \varepsilon' \between x,\ y)^4 \\ G &= (\alpha'',\ \tfrac{1}{8}\beta'',\ \tfrac{1}{28}\gamma'',\ \tfrac{1}{56}\delta'',\ \tfrac{1}{70}\varepsilon'',\ \ldots \between x,\ y)^8 \end{aligned}$$

we have

$$\begin{aligned}
&S, \text{ Coeff. } x^2 = a\varepsilon - b\delta + c\gamma - d\beta + e\alpha,\\
&T, \quad \text{"} \quad x^4 = a\delta - 2b\gamma + 3c\beta - 4d\alpha,\\
&U, \quad \text{"} \quad x^2 = 2\alpha'\delta - \beta'\gamma + \gamma'\beta - 2\delta'\alpha,\\
&V, \quad \text{"} \quad x^4 = 280\alpha''\varepsilon - 35\beta''\delta + 10\gamma''\gamma - 20\delta''\beta + 24\varepsilon''\alpha.
\end{aligned}$$

Similarly the invariant W and the leading coefficients of X, Y depend on the coefficients of A, G and E^3; and the invariant Z depends on the coefficients of G and E^4. But these two invariants W and Z have been already calculated; viz., as already mentioned, W is Salmon's invariant D, and Z his invariant E, given each of them in the second edition of his Higher Algebra (but not reproduced in the third edition): on account of the great length of these expressions it has been thought that it was not expedient to give them here.

For the reason appearing above, I have added the expressions for the remaining coefficients of C, E, G.

$A,\ x^6$	$B,\ x^0$	$C,\ x^4$	$D,\ x^8$	$E,\ x^2$	$F,\ x^6$	$G,\ x^8$	$H,\ x^{12}$
$a\ +1$	$ag\ +1$	$ae\ +1$	$ac\ +1$	$acg\ +1$	$ace\ +1$	$a^2f\ +1$	$a^2d\ +1$
	$a^0bf\ -6$	$a^0bd\ -4$	$a^0b^2\ -1$	$df\ -3$	$d^2\ -1$	$abe\ -5$	$abc\ -3$
	$ce\ +15$	$c^2\ +3$		$e^2\ +2$	$a^0b^2e\ -1$	$cd\ +2$	$a^0b^3\ +2$
	$d^2\ -10$			$a^0b^2g\ -1$	$bcd\ +2$	$a^0b^2d\ +8$	
				$bcf\ +3$	$c^3\ -1$	$bc^2\ -6$	
				$bde\ -1$			
				$c^2e\ -3$			
				$cd^2\ +2$			

$I,\ x^0$		$J,\ x^4$		$K,\ x^6$		$L,\ x^{10}$		$M,\ x^2$		$N,\ x^4$		$O,\ x^8$	
$aceg$	+ 1	a^2f^2	+ 1	a^2dg	+ 1	a^2cf	+ 1	a^2cg^2	+ 1	a^2cfg	— 1	a^2cdg	0
cf^2	— 1	$abef$	— 10	ef	— 1	de	— 1	dfg	— 6	deg	+ 1	cef	— 1
d^2g	— 1	cdf	+ 4	$abcg$	— 3	ab^2f	— 1	e^2g	+ 8	df^2	+ 3	d^2f	+ 3
def	+ 2	ce^2	+ 16	bdf	— 2	bce	— 2	ef^2	— 3	e^2f	— 3	de^2	— 2
e^3	— 1	d^2e	— 12	be^2	+ 5	bd^2	+ 4	ab^2g^2	— 1	ab^2fg	+ 1	ab^2dg	0
a^0b^2eg	— 1	a^0b^2df	+ 16	c^2f	+ 9	c^2d	— 1	$bcfg$	+ 6	$bceg$	+ 2	b^2ef	+ 1
b^2f^2	+ 1	b^2e^2	+ 9	cde	— 17	a^0b^3e	+ 3	$bdeg$	— 34	bcf^2	— 3	bc^2g	0
$bcdg$	+ 2	bc^2f	— 12	d^3	+ 8	b^2cd	— 6	bdf^2	+ 48	bd^2g	— 4	$bcdf$	— 14
$bcef$	— 2	$bcde$	— 76	a^0b^3g	+ 2	bc^3	+ 3	be^2f	— 18	$bdef$	— 12	bce^2	+ 11
bd^2f	— 2	bd^3	+ 48	b^2cf	— 6			c^2eg	+ 18	be^3	+ 15	bd^2e	+ 1
bde^2	+ 2	c^3e	+ 48	b^2de	+ 2			c^2f^2	— 45	c^2dg	+ 1	c^3f	+ 9
c^3g	— 1	c^2d^2	— 32	bc^2e	+ 6			cd^2g	+ 4	c^2ef	+ 9	c^2de	— 14
c^2df	+ 2			bcd^2	— 4			$cdef$	+ 78	cd^2f	+ 4	cd^3	+ 6
c^2e^2	+ 1							ce^3	— 36	cde^2	— 21	a^0b^3cg	0
cd^2e	— 3							d^3f	— 48	d^3e	+ 8	b^3df	+ 8
d^4	+ 1							d^2e^2	+ 28	a^0b^3eg	— 3	b^3e^2	— 9
								a^0b^2ceg	0	b^2cdg	+ 6	b^2c^2f	— 6
								b^2d^2g	+ 64	b^2cef	+ 9	b^2cde	+ 16
								b^2def	— 144	b^2d^2f	+ 32	b^2d^3	— 8
								b^2e^3	+ 81	b^2de^2	— 39	bc^3e	— 3
								bc^2dg	— 96	bc^3g	— 3	bc^2d^2	+ 2
								bc^2ef	+ 108	bc^2df	— 66		
								bcd^2f	+ 96	bc^2e^2	+ 18		
								$bcde^2$	— 126	bcd^2e	+ 76		
								bd^3e	+ 16	bd^4	— 32		
								c^4g	+ 36	c^4f	+ 27		
								c^3df	— 72	c^3de	— 45		
								c^3e^2	— 27	c^2d^3	+ 20		
								c^2d^2e	+ 96				
								cd^4	— 32				

	P, x^0
$a^2d^2g^2$	$+1$
$defg$	-6
df^3	$+4$
e^3g	$+4$
e^2f^2	-3
$abcdg^2$	-6
$bcefg$	$+18$
bcf^3	-12
bd^2fg	$+12$
bde^2g	-18
be^3f	$+6$
c^3g^2	$+4$
c^2e^2g	-24
c^2dfg	-18
c^2ef^2	$+30$
cd^2eg	$+54$
cd^2f^2	-12
cde^2f	-42
ce^4	$+12$
d^4g	-20
d^3ef	$+24$
d^2e^3	-8
$a^0b^3dg^2$	$+4$
b^3efg	-12
b^3f^3	$+8$
$b^2c^2g^2$	-3
b^2ce^2g	$+30$
b^2cef^2	-24
b^2d^2eg	-12
$b^2d^2f^2$	-24
b^2de^2f	$+60$
b^2e^4	-27
bc^3fg	$+6$
bc^2deg	-42
bc^2df^2	$+60$
bc^2e^2f	-30
bcd^3g	$+24$
bcd^2ef	-84
$bcde^3$	$+66$
bd^4f	$+24$
bd^3e^2	-24
c^4eg	$+12$
c^4f^2	-27
c^3d^2g	-8
c^3def	$+66$
c^3e^3	-8
c^2d^3f	-24
$c^2d^2e^2$	-39
cd^4e	$+36$
d^6	-8

	Q, x^6	R, x^6
a^3dg^2		-1
efg		$+9$
f^3		-8
a^2bcg^2		$+3$
$bdfg$		-24
be^2g		-45
bef^2		$+66$
c^2fg	-2	$+3$
$cdeg$	$+5$	$+48$
cdf^2	$+6$	-12
ce^2f	-7	-51
d^3g	-3	-16
d^2ef	-3	$+36$
de^3	$+4$	-8
ab^3g^2	0	-2
b^2cfg	$+4$	$+12$
b^2deg	-5	$+192$
b^2df^2	-6	-48
b^2e^2f	$+7$	-144
bc^2eg	-5	-159
bc^2f^2	-6	$+18$
bcd^2g	$+7$	-48
$bcdef$	-16	$+24$
bce^3	$+23$	$+279$
bd^3f	$+30$	-48
bd^2e^2	-33	-84
c^3dg	-1	$+42$
c^3ef	$+36$	$+153$
c^2d^2f	-37	-36
c^2de^2	-53	-399
cd^3e	$+79$	$+312$
d^5	-24	-64
a^0b^4fg	-2	0
b^3ceg	$+5$	0
b^3cf^2	$+6$	0
b^3d^2g	$+2$	-224
b^3def	$+22$	$+144$
b^3e^3	-27	$+54$
b^2c^2dg	-8	$+336$
b^2c^2ef	-39	-108
b^2cd^2f	-50	$+384$
b^2cde^2	$+107$	-684
b^2d^3e	-22	$+144$
bc^4g	$+3$	-126
bc^3df	$+84$	-648
bc^3e^2	-21	$+432$
bc^2d^2e	-102	$+564$
bcd^4	$+44$	-288
c^5f	-27	$+270$
c^4de	$+45$	-450
c^3d^3	-20	$+200$

Remaining Coefficients of C, E, G.

C

x^3y	
af	$+\ 2$
be	$-\ 6$
cd	$+\ 4$

x^2y^2	
ag	$+\ 1$
ce	$-\ 9$
d^2	$+\ 8$

xy^3	
bg	$+\ 2$
cf	$-\ 6$
de	$+\ 4$

y^4	
cg	$+\ 1$
df	$-\ 4$
e^2	$+\ 3$

E

xy	
adg	$+\ 1$
aef	$-\ 1$
bcg	$-\ 1$
bdf	$-\ 8$
be^2	$+\ 9$
c^2f	$+\ 9$
cde	$-\ 17$
d^3	$+\ 8$

y^2	
aeg	$+\ 1$
af^2	$-\ 1$
bdg	$-\ 3$
bef	$+\ 3$
c^2g	$+\ 2$
cdf	$-\ 1$
ce^2	$-\ 3$
d^2e	$+\ 2$

G

x^7y	
a^2g	$+\ 1$
abf	$+\ 2$
ace	$-\ 19$
ad^2	$+\ 8$
b^2e	$-\ 6$
bcd	$+\ 44$
c^3	$-\ 30$

x^6y^2	
abg	$+\ 7$
acf	$-\ 14$
ade	$-\ 14$
b^2f	0
bce	$-\ 21$
bd^2	$+\ 112$
c^2d	$-\ 70$

x^5y^3	
acg	$+\ 7$
adf	$-\ 28$
ae^2	$-\ 14$
b^2g	$+\ 14$
bcf	$-\ 42$
bde	$+\ 168$
c^2e	$-\ 105$

x^4y^4	
adg	0
aef	$-\ 35$
bcg	$+\ 35$
bdf	0
be^2	$+\ 105$
c^2f	$-\ 105$

x^3y^5	
aeg	$-\ 7$
af^2	$-\ 14$
bdg	$+\ 28$
bef	$+\ 42$
c^2g	$+\ 14$
cdf	$-\ 168$
ce^2	$+\ 105$

x^2y^6	
afg	$-\ 7$
beg	$+\ 14$
bf^2	0
cdg	$+\ 14$
cef	$+\ 21$
d^2f	$-\ 112$
de^2	$+\ 70$

xy^7	
ag^2	$-\ 1$
bfg	$-\ 2$
ceg	$+\ 19$
cf^2	$+\ 6$
d^2g	$-\ 8$
def	$-\ 44$
e^3	$+\ 30$

y^8	
bg^2	$-\ 1$
cfg	$+\ 5$
deg	$-\ 2$
df^2	$-\ 8$
e^2f	$+\ 6$

Note.—In the tables on this page, a has been treated like the other letters; on the preceding pages, the powers of a have been suppressed except in the first of every series of terms containing a common power of a.

The final result is that we have the values of the invariants B, I, P, W, Z and the leading coefficients of the covariants A, C, D, E, F, G, H, J, K, L, M, N, O, Q, R: also the means of calculating the leading coefficients of the remaining covariants S, T, U, V, X, Y.

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